ANL/NDM 156

A Demonstration of the Lognormal Distribution

by

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ABSTRACT

Although its applicability is not universal, the lognormal distribution is frequently used to represent physical parameters that are inherently positive and are not well known, i.e., that have large errors. In particular, arguments drawn from Bayesian statistics suggest that the lognormal function is the optimal choice of a probability distribution for those cases where the only knowledge possessed about a particular parameter is an estimate of its mean value and associated error. In this report it is demonstrated by a simulation exercise that the lognormal function represents very well the outcomes of repeated measurements that are subjected to a variety of multiplicative disturbances that, by their very nature, preclude the observation of negative values. In the present study, a simplified model of measurement is defined and it is then subjected to numerical analysis using the Monte Carlo method. There is no attempt to prove rigorously that the lognormal distribution results as the unique consequence of assumptions about the defined model, nor are other more complex models of measurement investigated. In any event, it is not possible to produce a proof by the use of anecdotal information, e.g., by analyzing the results of repeated Monte Carlo trials. Nevertheless, the simulation study described here does provide strong evidence that for many practical situations the lognormal distribution can be used to represent the probable outcome of measurements of positive quantities.